International Journal of Theoretical Physics, Vol. 16. No. 7 (1977), pp. 555–560

Letter

Comments on Fu's Paper on the Clock Paradox as a Cosmological Problem

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Received: 11 *April* 1977

In a recent article in this journal (Fu, 1976), the claim was made that asymetric aging is predicted by the theory of general relativity, as a consequence of the interactions between clocks and the rest of the universe. The claim was also made that because of this conclusion, my earlier proof (Sachs, 1971) that asymmetric aging is not predicted by the theory of relativity, *per se*, is neither rigorous nor technically valid.

Contrary to Fu's claim, my proof was indeed rigorous – it did not entail any models or approximations. It was based on the most general expression of this theory that would be consistent with its underlying symmetry group. Neither was Fu's contention correct that I had a "different view of $g_{\alpha\beta}$" than general relativity. What I *did* emphasize in my papers was that the metric relations between the points of space-time must *necessarily* be more general than the original 10-component symmetric tensor formulation of the metric tensor field – because the underlying symmetry group of general relativity theory is broader than is indicated by the symmetric tensor representation. However, I did not claim that the metric field should be interpreted in a way different from its original meaning according to Einstein's theory – just as Dirac's generalization of the Schrödinger wave function, giving the spinor form, did not imply that Dirac was then giving a different *meaning* to wave mechanics!

My proof that the theory of general relativity does not predict asymmetric aging was based on a rigorous demonstration that $\oint ds = 0$ for the most general representation for ds, associated with the curved space-time of general relativity theory (Sachs, 1971). Further comments on this analysis were given in later papers (Sachs, 1973a, 1973b, 1974). This was a functional analysis, not dependent on anything other than the general features of space-time – *as it relates to matter*, according to the full meaning of general relativity theory.

At the beginning of his paper, Fu stated the basis of his objection to my proof: "the closed path line integral in Sachs' argument is not rigorous because a closed path cannot be arbitrarily drawn." He then went on to say that this is because "its physical significance in the problem of the clock paradox exists only in a space-time with an intrinsic property of rotation."

Fu's statement is false because my closed path was *not* drawn arbitrarily! With the unique representation of the solutions (the metric field components), there is a unique geodesic path predicted. This, in turn, is the solution of the geodesic equation. Recall that the geodesic path, $\oint ds$, according to general relativity theory, is in principle unique in the exact expression of the theory – that is, when it entails all of the matter of a closed system, at the outset (Sachs, 1976). One loses uniqueness in the predictions of the formalism only when a part of the description, representing particular physical interactions, is artificially removed, for practical reasons in particular problems.

My closed-path integral,

$$\oint ds \equiv \oint_{1\,s_1}^{s_2} ds - \oint_{2\,s_1}^{s_2} ds$$

involves two different geodesic paths, C_1 and C_2, because of the different physical conditions that determine them, *according to Einstein's theory*. For example, C_1 might refer to a twin brother sitting here on Earth, watching his brother fly away in a rocket ship at some high speed, The matter field, which is the source (the right-hand side) of the metrical field equations, then predicts the contour C_1 *uniquely*. But the path C_2 (of the "traveler") is different – because his matter field source, determining the metrical field, is different. The matter field source for him must incorporate the "blast-off" of his rocket ship, his interaction with other planets on his trip, his interaction with his ship when it turns around to come home, etc. That is, C_1 and C_2 are two different *exact* geodesics. They are different because the physics of the matter fields that uniquely determined the corresponding metric fields was (unambiguously) different in the two cases (Sachs, 1973b). (Of course, I am assuming here that the brothers themselves do not exert forces on each other to affect their motions.) In the clock problem, then, the twin brothers (or any other physically identical mechanisms) are initially and finally in the same inertial frame of reference, and at rest relative to each other, at the respective space-time points s_1 and s_2. In between these space-time points, they are in different (though uniquely determined) relatively moving frames of reference. Thus, there is no ambiguity in the closed path that I consider in my analysis.

Fu's claim that the closed path must relate to two separate space-time systems is entirely false, with respect to the role played by space-time in the theory of general relativity. And his claim that the "backward path," $-\int ds$, is the time reversal of the "forward path," $+\int ds$, is false within the quaternion metrical field description that I use – which he also claims to refer to in his analysis. In this case, $ds = q_\alpha \, dx^\alpha$. But the quaternion field, q_α, is not covariant under reflections in time (or space) – as the differential dt would be, in the real number formulation. The time reversal of a quaternion field is its *conjugate field*: $\tilde{q}_\alpha \, dx^\alpha$ (Sachs, 1967).

Further, the path characterized by $\oint ds$ in my analysis has nothing to do with the fact that the space-time has rotation. In fact, going from the 10-component symmetric tensor field metrical field, $g_{\alpha\beta}$, to the 16-component metrical field, q_α, automatically introduces a torsion into the space-time. But this *implicit* torsion does not have to be exploited in the *explicit* analysis of the clock problem – just as an analysis of the coupling of an electron to an electric field does not exploit the electron's spin degrees of freedom.

While, in his paper, Fu says that he will treat the clock problem in terms of the quaternion form, $ds = q_\alpha \, dx^\alpha$, he does not actually do this since he still treats ds as a real number field. Fu (correctly) says that the extremum condition, $\delta \int ds = 0$, leads to the geodesic equation; but he does not seem to realize that the derivatives with respect to ds in the latter equation are "quaternion derivatives" – not real number derivatives! Thus, what he takes as a single (real number) equation of motion – the geodesic equation – is in this theory a quaternion equation, expressible in terms of *four* real number equations. The geodesic equation then entails a *quaternion calculus*, rather than a real number (or a complex number) calculus (Sachs, 1970, 1975). This generalization was crucial in my analysis of the clock problem (Sachs, 1971).

According to general relativity theory, ds is an abstract entity, whose invariance properties prescribe the covariance of the laws of nature. ds is *not* the physical reading of a clock! In my analysis, I extracted the reading of a clock (of any sort – a human being's heart beat, the duration of a block on an inclined plane, the decay time of mesons, etc.) from the abstract quaternion differential element, $ds = q_\alpha \, dx^\alpha$. It was argued that if ds is to be calibrated with some correlation regarding the processes of the running of two identical clocks, then the equality of the different path integrals, $\oint_1 ds = \oint_2 ds$ for the respective clocks implies that there would be no asymmetric aging of one of these clocks with respect to the other. Thus, there is no logical paradox here! In Fu's (and the physics majority's) view, the logical paradox persists, so long as *motion* is defined to be a relative concept. In addition to this logical inconsistency, Fu is also mathematically inconsistent in his analysis that leads him to the conclusion that $\oint ds \neq 0$. He mistakenly uses the analysis of real number fields in the application to quaternion number fields, particularly in applying the theorems of the calculus of real functions in a Euclidean space to the calculus of quaternion functions in a Riemannian space.

It is true, as he says, that in ordinary calculus the closed path integral $\oint f_i dx^i = 0$ only if $f_i dx^i$ is an exact differential – which is the case if and only if $\partial_j f_i - \partial_i f_j = 0$. Nevertheless, q_α is a quaternion number field, rather than a real number field, and its calculus (differentiation, integration) does not correspond exactly with the real number calculus – just as the calculus of complex functions entails relations not encountered in real number analysis. For example, the "quaternion derivative" is not the same as the ordinary derivative of a function (Sachs, 1970, 1975). Further, the terms $q_{\alpha,\beta}$ are the *covariant derivatives* of the quaternion field in a *curved space-time* – these derivatives are then much more complicated and different from the terms $f_{i,j}$ in real number analysis (that Fu's results were based upon).

The main mathematical point of Fu's "proof" that $\oint ds \neq 0$ was based on an analysis in a special frame of reference – the comoving frame with the rotation of space-time. The question asked by Fu is whether or not $q_\alpha \, dx^\alpha$ is an exact differential, in which case the closed-path integral was asserted to be zero. He states that the latter corresponds to the necessary and sufficient condition that $q_{\alpha,\beta} = q_{\beta,\alpha}$. He states this as a theorem of calculus. But which calculus does he refer to? Are the derivatives in this relation the covariant derivatives or the ordinary derivatives of the quaternion fields. If they are the covariant derivatives, then the above relation is trivial – it is the tautology $0 = 0$, since all of the covariant derivatives of the quaternion metrical field vanish in all frames of reference – by definition. (This is analogous to the vanishing of the covariant deratives of the metric tensor, in all frames, in the standard expression of general relativity.)

If the "commas" in Fu's equation refer to the ordinary derivatives of the quaternion metric field (which it does refer to in the actual theorem of calculus that he refers to, in terms of real number fields) then it is certainly *not* true that this equality would hold.

Fu then goes on to say that "our proof, although based on the co-moving frame, is generally true since *ds* is Lorentz invariant." This is also false since the invariance in general relativity is with respect to the Einstein group, rather than the Poincaré group. Indeed, it is the *difference* between the representations of these two groups that is crucial in my analysis, leading to the mathematical part of the resolution of the clock problem.

Toward the end of his paper, Fu claimed that I imposed an undue restriction in requiring that the metrical field components should be analytic. But this restriction was not imposed by my theory in particular! It is generally imposed by the theory of general relativity, in its orthodox, exact formulation. Indeed, it is a feature of the differential geometry at the outset that the metrical field is continuous and continuously differentiable, everywhere, according to its use in facilitating a representation of an analytic, continuously variable matter source. Of course, one may object to this idea – but he must then admit that he is thereby objecting to the whole idea in general relativity of the relation between differential geometry and matter. On the other hand, if one is attempting to investigate the full set of implications of the theory of general relativity in the first place, then it is logically inconsistent to make this objection when one wishes to determine the actual implications of this theory (Sachs, 1973a)!

Fu's objection to my use of a special frame of reference in which a covariant function is analyzed in a two-dimensional slice of space-time was also empty. It is always permissible to take a special frame of reference in order to analyze covariant functions in this frame – so long as the final result is then globally extended to its general expression in the curved space-time.

Fu's claim of the need for boundary conditions to yield a rigorous solution in general relativity is also fallacious, if these conditions refer to smeared distributions of values of the field solutions at particular places and times. For in general relativity theory (especially so when it incorporates the Mach principle – which Fu claims to support), the boundary conditions, are after all, not more than a short-hand way of expressing the reactions of other matter to the matter described, and vice versa. If one is representing the features of a *closed system* at the outset, then the details of all of the matter fields in interaction makes the introduction of phenomenological boundary conditions redundant.

Finally, Fu says that "according to the Mach principle a certain kind of interaction must exist between the matter of the whole universe and a physical entity in the local domain." He then attributes the loss of synchronization of clocks in the asymmetric aging (which he claims *must* happen) to these "certain kinds of interactions." First, this is certainly a wrong statement of the "Mach principle". This principle does *not* say anything about new kinds of interactions, *per se*! It merely takes the interactions of matter as given – e.g., the infinite range Newtonian gravitational potential energy. What the principle does assert is that the *inertial mass* of any quantity of matter must relate to its interaction with other matter – whatever that interaction may be. The Mach principle is concerned with the origin of the inertia of matter in interaction, not with the form of the interaction that is exerted.

With Fu's view, he then appeals to Mach for a new (so far, mysterious) force to be exerted by the distant stars on clocks that would move relative to an observer, to make them go slowly compared with the observer's clocks. Not only is this claim antirelativistic – since it appeals to a special category for the "observer" – but it certainly would have been logically and scientifically rejected by Mach, because of the "method of science" that it appeals to. This is because Fu's claim is that an effect certainly happens (the asymmetric aging) because of his faith that a certain kind of interaction (yet undiscovered) must exist! But in science, one must demonstrate precisely what is the explicit nature of this interaction, precisely what it does to other matter – dynamically and in mathematical terms, and precisely what are the mathematical relations that predict this interaction. Until this can be accomplished, Fu's claim remains mystical, rather than scientific!

In conclusion, I do not believe that Fu resolved the logical paradox of the clock problem, nor that he proved that $\oint ds \neq 0$ for a closed path of a Riemannian space-time – i.e., he did not prove that the path length of a geodesic in a Riemannian space-time is path-dependent. Without any actual proof, and in the face of the logical paradox in his conclusion, Fu reflects that his final conclusion is nevertheless in agreement with the majority in physics today. However, one needn't dig very deeply into the history of science to learn of the error in judgement that consensus of opinion can be a bona fide criterion for scientific truth!

Fu wonders why philosophers may think that asymmetric aging is hardly understandable. He does not seem to realize, however, that what it is that troubles the philosophers is that the predicted asymmetry – if it is really there! – *goes both ways*, according to the majority's interpretation of the space-time transformations of the theory of relativity. That is, there seems to be a prediction that one clock is both slow and fast compared with another clock! Indeed, one need not be a schooled philosopher to realize that a logical paradox, such as this one, cannot express something that is *real*!

Fu's concluding comment was: "A great life lives longer because of its stronger cosmological sense." I agree with this. But I would add that: Though a man may think he has a cosmological sense, this is only an illusion so long as it is without reason!

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